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# Diaphragm Couplings Versus Gear Couplings for Marine Applications

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# Diaphragm Couplings Versus Gear Couplings for Marine Applications

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**Abstract**

The gear coupling has been in existence for over 50 years. As advances in marine propulsion have occurred, so have advances in gear couplings. There are many variables in gear couplings that can affect their characteristics. Including tooth design, materials, and lubrication methods. All couplings react on connected equipment. A system designer must consider these reactions when designing a system. If a gear coupling is chosen, there are many characteristics which are difficult to predict: therefore, one must conservatively estimate the maximum forces and moments that can be anticipated. This usually will make the system rather large and heavier than may be required. The diaphragm coupling usually has more predictable coupling characteristics, which can make a designer's life easier. This paper compares the characteristics of diaphragm couplings versus the gear (dental) type couplings in marine applications. Applications of couplings for main propulsion and auxiliary equipment are discussed. The methods used to analyze the design and calculate the forces and moments generated by both the gear coupling and the diaphragm coupling are also provided. These analyses are used to show that the forces and moments generated by a diaphragm coupling are not only predictable, but are usually lower than those of a gear coupling. The paper shows that a diaphragm coupling can provide a more predictable, reliable alternative to the gear coupling for advanced marine applications.

In today's technology, higher horsepowers and higher speeds continue to advance the art of rotating equipment design. This is especially evident in new marine propulsion technology where reliability, weight, noise, maintenance, and reaction loads on system components are of greater importance. Both the diaphragm coupling and the dental tooth coupling can be used for these high-performance applications. When selecting couplings for marine applications the system designer must look at the inherent design characteristics of the candidate couplings. Consideration should also be given to how well the flexible coupling will accommodate the application requirements without unleashing forces and moments that become very difficult to handle.

Some of these examples include main propulsion couplings for lineshafting, intermediate-speed gears and high-speed gears. Couplings that would be used for auxiliary equipment such as pumps and generators are also some of the more common applications.

All couplings react on connected equipment components when subjected to misalignment and torque. These reaction forces are greater for some couplings than others and if not considered can cause failure of shafts, bearings, and other equipment components. When these reactions are large or unpredictable the impact on the system is usually increased size and weight.

#### Types of Couplings

Couplings can be basically categorized as one of two types, the rigid coupling and the flexible coupling. Rigid couplings (Fig. 1) are usually used to connect equipment that experiences very small misalignments. Since rigid couplings also produce the greatest reaction on connected equipment, most applications require the use of flexible couplings.

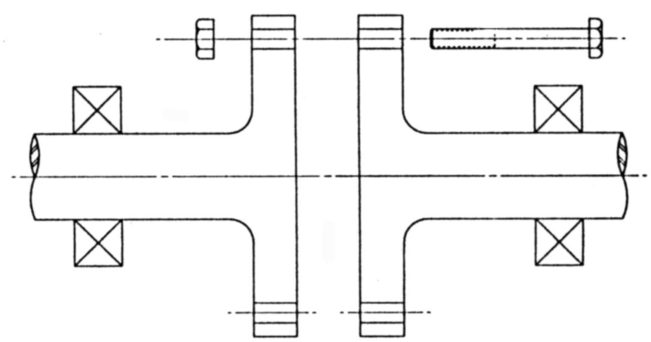


Fig. 1 Rigid Coupling

Flexible couplings are categorized as one of four types:

1. Mechanical flexible
2. Elastomeric
3. Metallic membrane
4. Miscellaneous

For the applications being compared, a mechanically flexible gear coupling or a metallic membrane coupling are most often considered because of torque, misalignment, speed, and environmental requirements. The gear coupling is a type of mechanically flexible coupling and the diaphragm is a type of metallic membrane coupling. Both types of couplings have unique characteristics that are suitable for the applications being considered.

**Gear couplings**. A gear coupling (Fig. 2) accommodates misalignment and shaft end float by the misalignment and sliding of the gear teeth of the hub in the sleeve (Fig. 3). Due to this mechanical motion, these couplings must be lubricated (unless one of the moving parts is made of material that supplies its own lubrication such as nylon). The most common mode of failure for a gear coupling is wear. It is one of the most common and simplest couplings used

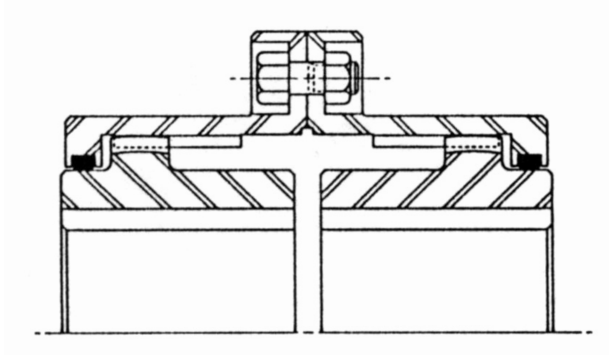


Fig. 2 Typical Gear Coupling

today. Due to the number of variables that can affect its successful operation it is usually difficult to design and evaluate. Some of the variables affecting its design and characteristics are:

1. Tooth Design
   - A. Straight or type and amount of crown
   - B. Pressure angle of tooth
   - C. Amount of backlash
   - D. Accuracy of tooth spacing
2. Material
   - A. Type of material(s)
   - B. Type of core heat treatment
   - C. Type of surface treatment
3. Lubrication
   - A. Oil
   - B. Grease
   - C. Sealed lubrication
   - D. Continuous lubrication
   - E. Method of lubrication

A gear coupling has its most significant effect not only on itself, but on system components from the forces and moments generated when it slides and/or misaligns. When a gear coupling accommodates the shaft float from thermal growth, hull deflection, shock, etc., axial forces react back onto the thrust bearings and other equipment. When misaligned a gear coupling will produce a bending moment that will load equipment shafts, bearings, and other system components. Both the axial forces and bending moments are significantly affected by the lubrication and the coefficient of friction between the mating gear teeth members.

**Diaphragm couplings.** A diaphragm coupling (Fig. 4) is one type of metallic membrane coupling. A diaphragm coupling consists of one or more metallic membranes which are attached at the outside diameter of a drive flange and transfers torque radially through the diaphragm to an inside diameter attachment. The other type of metallic membrane coupling is the disk coupling.

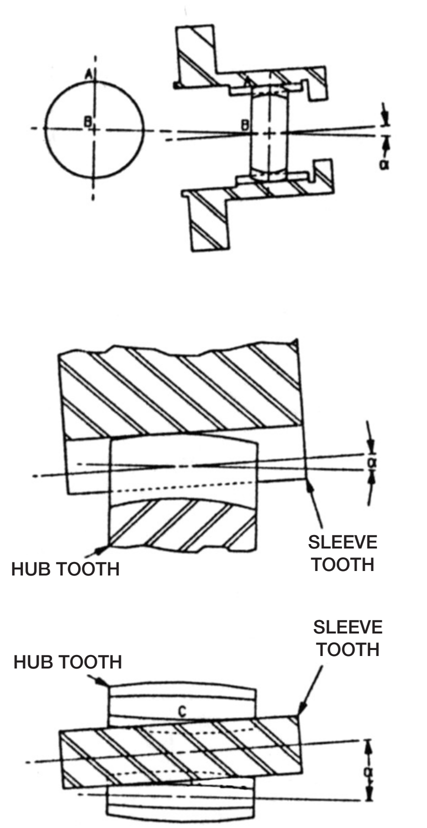



Fig. 3 Misalignment of a crown tooth coupling

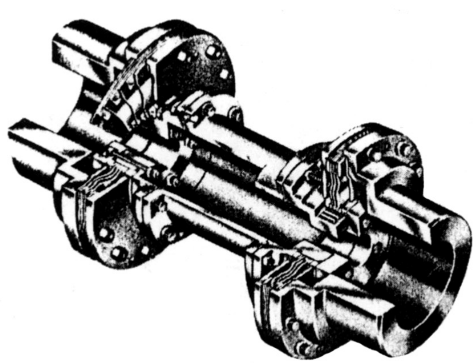


Fig. 4 Diaphragm coupling

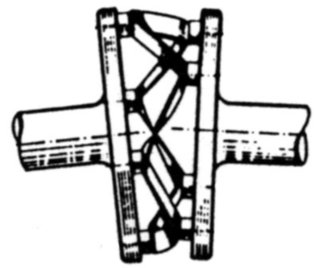


Fig. 5 Disc Coupling

A disk coupling will usually consist of several flexible metallic membranes which are alternately attached with bolts to opposite flanges (Fig. 5).

Metallic membrane couplings were developed because of a desire to eliminate the problems associated with a lubricated coupling. Their use became widespread in the late 1960's. A metallic membrane coupling relies on the flexure of metallic materials to accommodate axial travel and misalignment of equipment shafts. The axial force imposed on bearings and equipment is a factor of the deflection imposed on the diaphragm and the axial stiffness of the diaphragm flex unit. The bending moment that is transferred to the shafts, bearings, etc., is a function of the diaphragm bending stiffness of the flex unit and the amount of misalignment imposed.

### Couplings in Marine Propulsion

A typical propulsion system. An example of a modern marine propulsion gear is a double-reduction or dual torque path system (Fig. 6). Two turbines are used which are connected to their first reduction pinions by means of a high-speed flexible coupling. Each of the two first reduction pinions meshes with a pair of first reduction gears which then drive four second reduction pinions through intermediate-speed couplings. The second reduction pinions will next drive the main bull gear which is connected to the lineshaft by a low-speed coupling.

Auxiliary equipment such as pumps can be driven by the intermediate-speed pinions.

**High-speed coupling.** The high-speed coupling connects the turbine to the first reduction pinion. This application typically has moderate torque, moderate misalignments, and high axial travels. The high-speed coupling operates at speeds up to 6000 rpm. Figures 7 and 8 show a dental tooth coupling and Fig. 9 shows a diaphragm coupling that may be used for this application.

**Intermediate-speed coupling.** The first reduction gears drive the second pinions through an intermediate-speed coupling which can be mounted to a quill shaft. Moderate torque, low misalignments, and moderate axial travels are usually seen as operating requirements. The operating speed ranges up to 3000 rpm. Figure 10 shows a typical dental

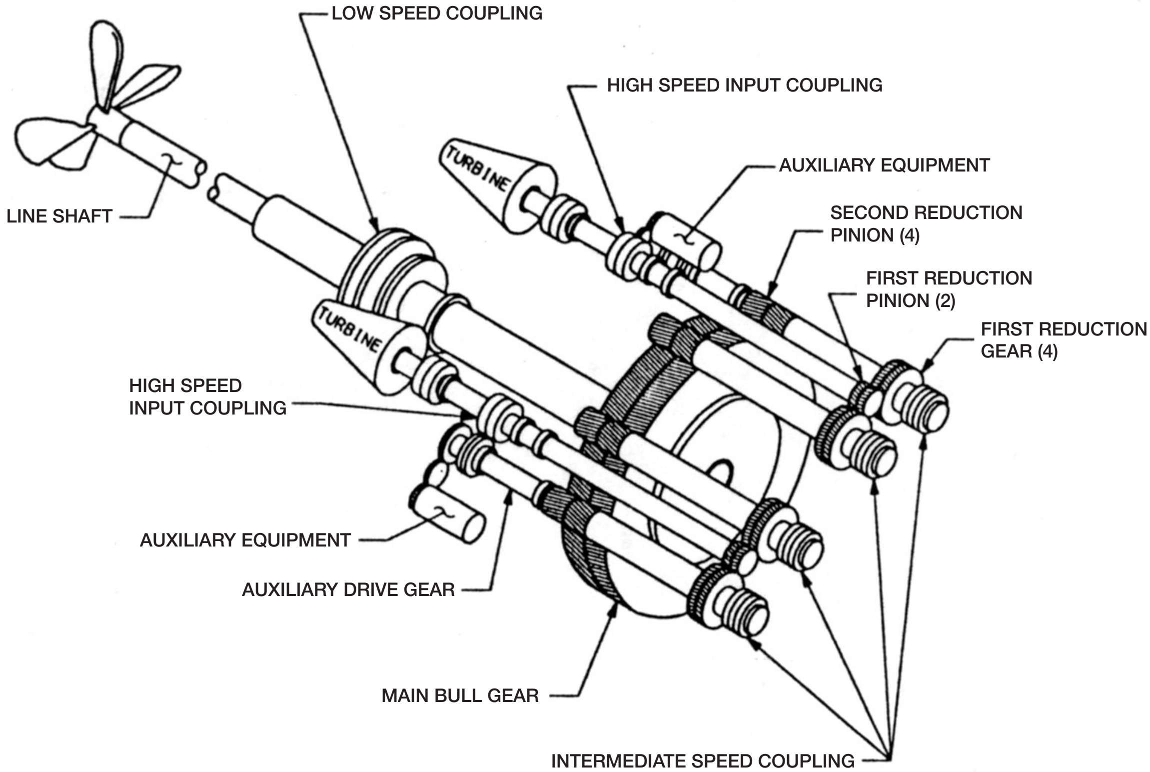



Fig. 6 Typical propulsion system

tooth coupling and Fig. 11 a typical diaphragm coupling.

**Low-speed coupling**. The low-speed coupling connects the main bull gear to the lineshaft, usually through a quill shaft. Operating conditions are high torques, moderate misalignments and moderate axial travels with very low rotational speeds. The operating speeds could vary from a few rpm to approximately 200 rpm maximum.

A low-speed coupling will typically consist of a diaphragm (Fig. 12) or gear coupling (Figs. 13 and 14) combined with a noise attenuation coupling. A noise attenuation coupling is an elastomeric coupling which uses an elastomer (rubber or neoprene) in compression and/or shear to transfer torque and sometimes misalignments while reducing system noise.

## Gear Coupling Design Characteristics

**Gear coupling characteristics**. Gear couplings can be classified in many ways, but the most common classification is by the type of tooth form. All gear couplings have straight-sided internal gear teeth. These internal teeth are involute formed, usually at a 20-deg pressure angle. Most of the backlash required to misalign a gear coupling is generally cut into the internal teeth. The two types of hub tooth form used to classify gear couplings are:

1. Straight sided (as described above—Fig. 15)
2. Crowned tooth (Fig. 16)

A crowned tooth is crown hobbed or shaped with a cutter on a cam. For small misalignment angles of $^1/_8$ deg or less, a straight tooth works almost as well as a crowned tooth. For angles greater than $^1/_8$ deg or less, a straight tooth should be used as the crown allows greater tooth contact during misalignment which reduces stresses.

One of the most important considerations in the design of gear couplings is the amount of heat generated by the gear under load and misalignment. There have been many attempts

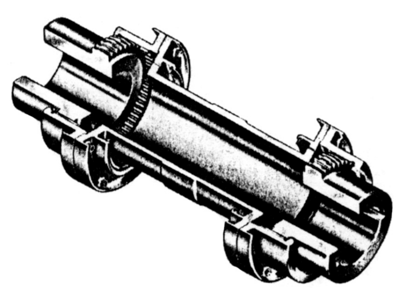


Fig. 7 High-performance gear coupling

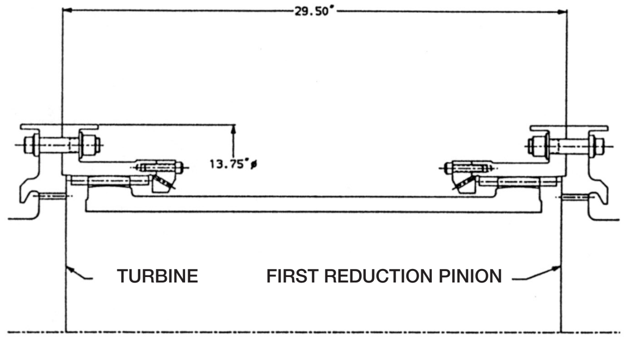



Fig. 8 High-speed coupling (gear type)

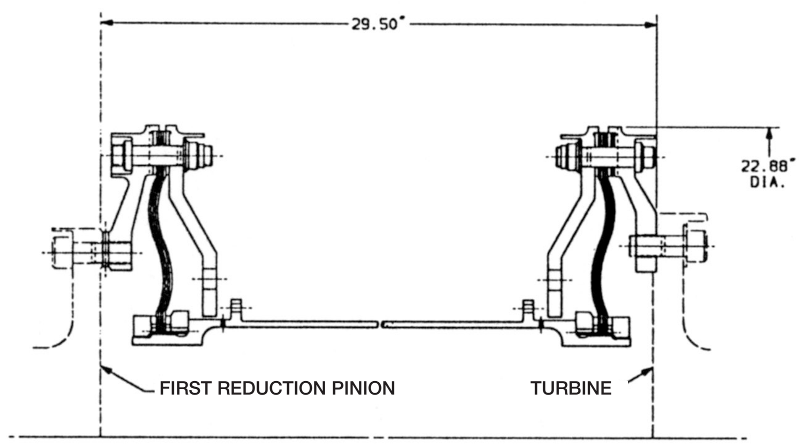



Fig. 9 High-speed coupling (diaphragm type)

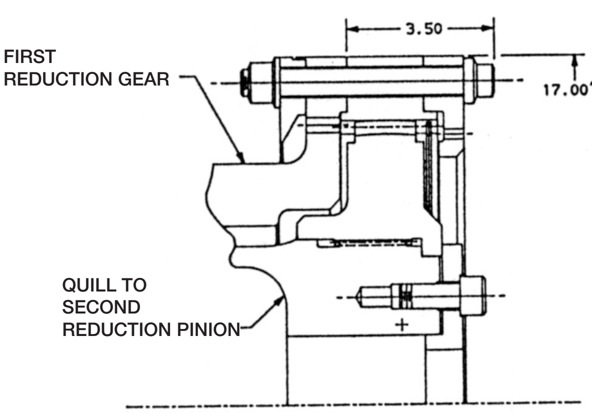



Fig. 10 Intermediate-speed coupling (gear type)

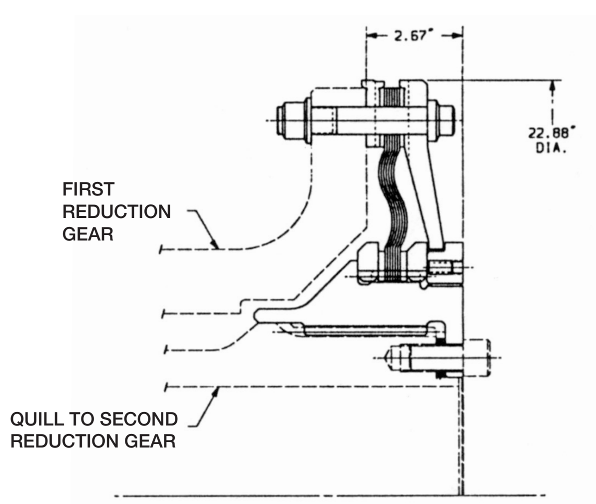



Fig. 11 Intermediate-speed coupling (diaphragm type)

Fig. 12 Low-speed coupling (diaphragm type with pin and bushing)

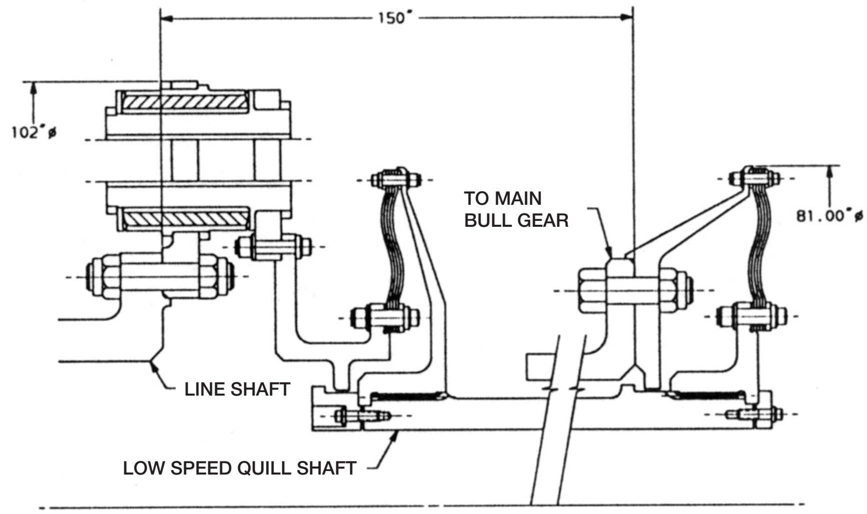



Fig. 13 Gear coupling with pin and bushing

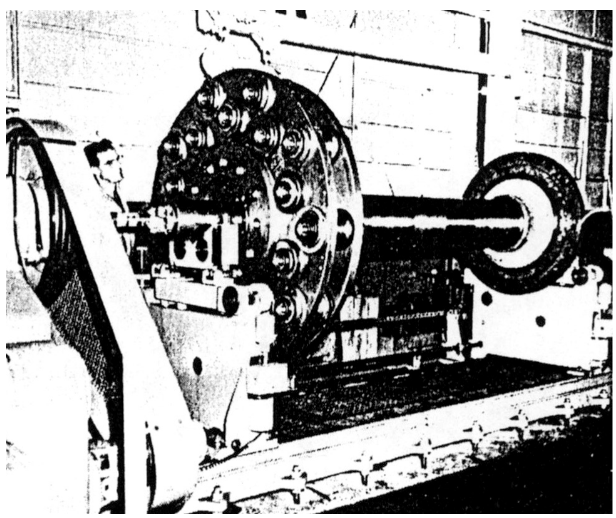
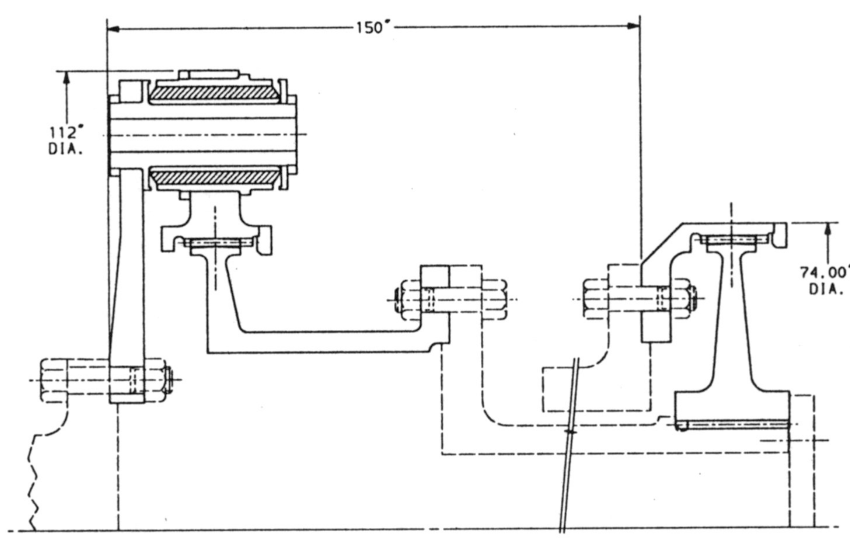


Fig. 14 Low-speed coupling (gear type with pin and bushing)



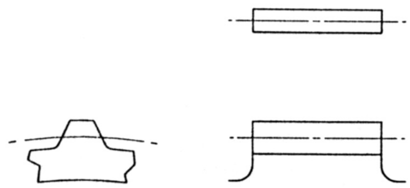


Fig. 15 Straight sided hub tooth

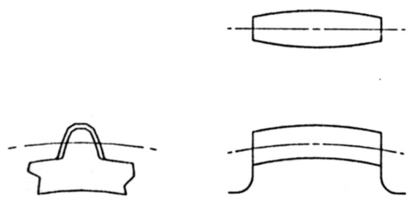


Fig. 16 Crowned hub tooth

to establish limits and criteria. The most commonly used is compressive stress or sliding velocity, but neither has been proven to be sufficient. Testing and experience have shown that design criteria for the teeth of gear couplings should consider the following:

1. Load
2. Misalignment
3. Tooth geometry (pitch diameter, diametral pitch, amount of tooth crown)
4. Percent of teeth in contact
5. Speed
6. Material, hardness of teeth, and finish
7. Type and quality of lube.

See Appendix 1 for an analysis of compressive stress and sliding velocity criteria.

Table 1 Compressive stress/sliding velocity comparison

| | |
|---|---|
| Material | AISI 1045 |
| Lubrication method | Grease packed |
| Misalignment | 3/4 Degrees |
| ScV (PSI-IPS) | 1,500,000 |
| Material | AISI 4140 with nitrided teeth |
| Lubrication method | Continuously lubed |
| Misalignment | 1/4 Degrees |
| ScV (PSI-IPS) | 750,000 |
| Material | Nitralloy N with nitrided teeth |
| Lubrication method | Grease packed |
| Misalignment | 4-1/2 Degrees |
| ScV (PSI-IPS) | 6,000,000 |
| Material | AISI 3310 with carburized teeth |
| Lubrication method | Grease packed |
| Misalignment | 3 Degrees |
| ScV (PSI-IPS) | 500,000 |

The compressive stress/sliding velocity criterion ($S_cV$) can be used to compare equivalent couplings and applications. Limits on $S_cV$ depend on the type of lubrication and the material used for a particular coupling. Some typical values based on testing and field experience are given in Table 1. It should be remembered that the allowable values change with material, tooth geometry, tooth finish, and type of lubrication.

### How Gear Couplings Affect the System

The most significant effect that a gear coupling has on the system comes from the moments and forces generated when it slides and/or misaligns.

1. *Axial force*—The formula for the axial force reacting on the thrust bearings and other system components is

$$\text{Force} = \frac{T \times \mu}{R} = \text{lb}$$

where

$T$ = Torque (in.-lb.)
$\mu$ = Coefficient of friction
$R$ = Pitch radius of gear (in.)

Various coupling manufacturers use the following typical values from test data for the coefficient of friction:

Sealed lubricated gear couplings:
$\mu = 0.05$
Continuous lubricated gear couplings:
$\mu = 0.075$

Values of coefficient of friction higher than the above can be experienced although, if they are present for any period of time, the coupling is no longer flexible, which would more than likely precipitate the failure of one of its own components or some component of the coupled equipment. If a coupling is mechanically locked due to sludge or wear, the forces could be increased seven to eight times those normally expected.

There is still much discussion of how large a value to use for a safety factor when designing a system or even a gear coupling. Due to the high reliability necessary for critical naval propulsion applications, MIL-C-23233 requires gear couplings to be designed with the conservative coefficient of friction of $\mu = 0.25$ for continuous operating conditions.

2. *Bending moment*—The moments produced in a gear coupling will load equipment shafts and bearings (Fig. 17) and also change the operating characteristics of the equipment.

The three basic moments in a gear coupling as shown in Fig. 18 are:

- moment generated from transmitted torque and angle (rotated around the $Z$ - $Z$ axis),
- moment generated from frictional loading (rotated

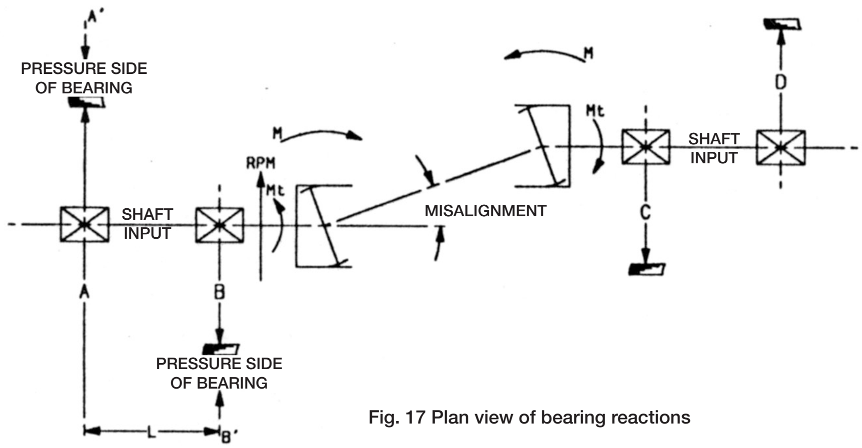



Fig. 17 Plan view of bearing reactions

around the *Z* - *Z* axis), and

- moment generated from displacement of the load from its center rotated around the *Y* - *Y* axis.

As in axial forces, the value of the coefficient of friction significantly affects the moments generated. See Appendix 1 for a detailed analysis of bending moments.

### Failure Mode for Gear Couplings

For gear couplings the most common type of failure is due to tooth wear and distress. Tooth distress is most commonly caused by the following:

1. Inadequate lubrication (Fig. 19).

2. Improper tooth contact (Fig. 20). In this case the matched lapped set was mixed. The resulting failure was not due to the tooth distress itself but to the increased bending moment which broke the coupling spacer (Fig. 21).

3. Worm tracking. Cold flow or welding occurs more frequently on continuous lubed couplings. This occurs near the end of the teeth when misalignment approaches the design limit of the crown and the lubrication film breaks down and causes metal-to-metal contact. High, localized tooth loading or lubricant deterioration can cause this type of failure as described in No. 2 above.

4. Sludge buildup can cause failure of almost any system component such as shafts or bearings (Fig. 22). Sludge also will collect corrosive residue, which can corrode coupling parts and act as a source of crack initiations for a fatigue-propagated failure of a part.

### Diaphragm Coupling Design Considerations

As stated earlier, there are two basic types of flexible metallic membrane couplings, the diaphragm and the disk. Either type can be used singularly or as a pair separated by a spacer or spool piece.

Metallic membrane coupling use was limited through the 1940's to low-torque, low-speed applications where only limited amounts of misalignment were required. In the late 1940's the appearance of the small gas turbine produced the need for the thin contoured diaphragm, which saw usage in aircraft applications. The progress and acceptance of this coupling in industrial and marine applications was greatly hindered by the inability to accommodate high misalignment and large axial movements.

For many years gear couplings have been used on steam turbines, gas turbines, compressors, pumps and marine propulsion systems. When the horsepower, speeds, and operating temperatures increased, many problems with gear couplings developed. The need for lower moments, forces and noise production characteristics has pushed the advanced development and usage of diaphragm couplings in thousands of applications. Because of this developed technology, diaphragm couplings have been successfully used since the mid 1970's for propulsion systems.

Many manufacturers and users of rotating equipment

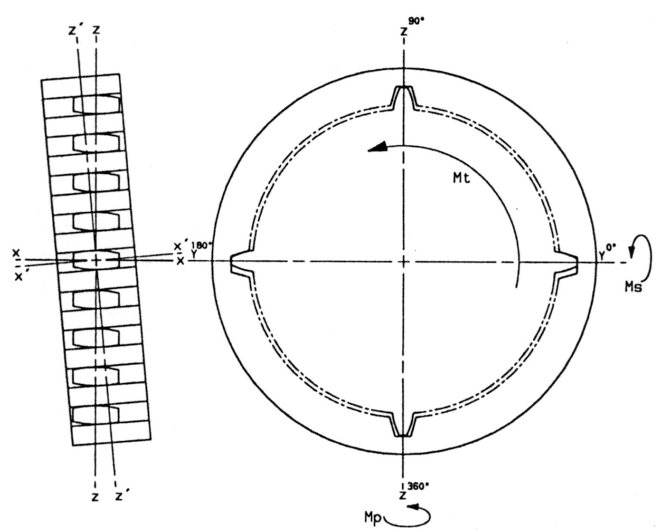



Fig. 18 Coordinate system for a gear coupling

have increased their list of coupling requirements to include the following;

- No lubrication;
- Higher torque capability without an increase in coupling size;
- Accommodation of greater misalignments;
- Accommodation of greater axial motions;
- Suitable for high-temperature operation;
- Not damaged by exposure to corrosive atmospheres;
- Adaptable to all types of connections: splines, taper shafts, flanges, etc.;
- Produce low moments and forces;
- Produce predictable moments and forces;
- Easily balanced;
- Operate for years without maintenance or problems;
- Produce low vibratory inputs into equipment.

Diaphragm couplings for these applications are available in three basic forms (Fig. 23):

- Tapered contoured,
- Multiple straight diaphragm with spokes,
- Multiple convoluted diaphragm.

All three shapes have some type of profile modification that helps reduce size, increase flexibility and control stress concentrations. For example, in order to increase flexibility, tapered contoured diaphragms are designed for a constant shear stress from the inner diameter to the outer diameter.

Couplings using multiple diaphragm designs, with

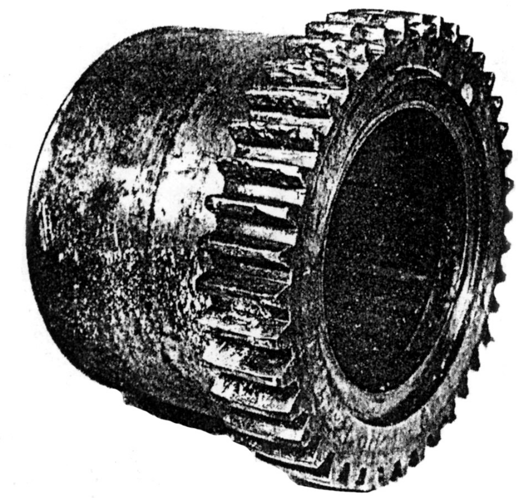


Fig. 19 Failure due to lubrication breakdown

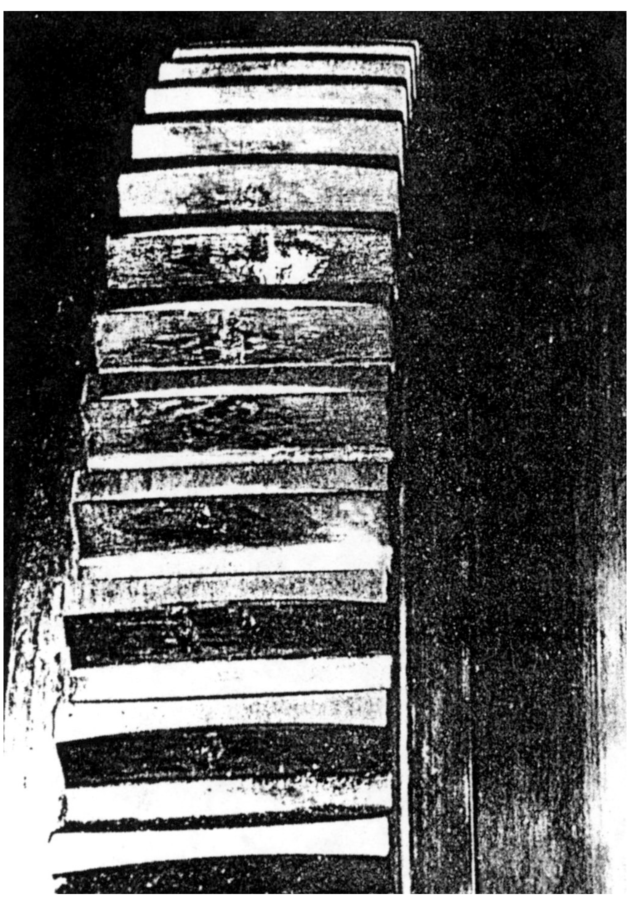


Fig. 20 Improper tooth contact

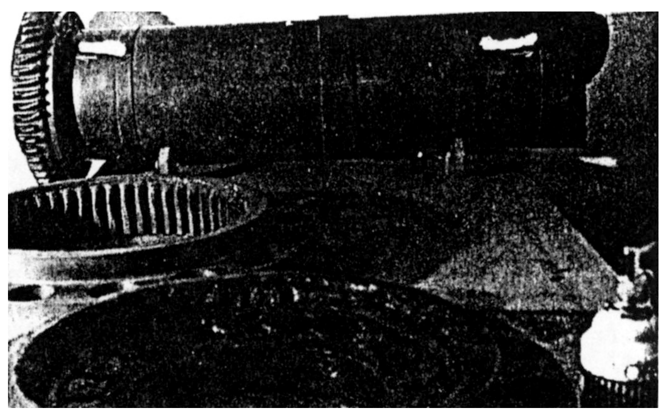


Fig. 21 Failure due to increased moment

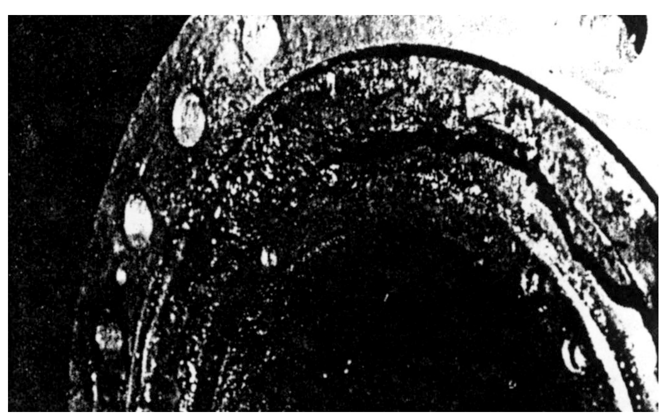


Fig. 22 Sludge contaminated gear coupling

a number of thin plates in parallel rather than a single thick one, have improved flexibility and usually lower stresses. Diaphragm stresses, moments and forces increase with the cube of the material thickness ($t^3$). Therefore, several thin diaphragms will produce lower stress values than a single thick one.

Since diaphragm couplings are usually used on high-performance equipment which requires high reliability, coupling components are usually made of high-strength alloys with good fatigue properties. Typical materials are AISI 4100 or 4300 steels coated for corrosion protection, PH stainless steels or high-strength nickel alloys.

Diaphragm couplings, like all metallic membrane couplings, are usually designed for infinite life. In applying membrane couplings to marine propulsion applications the most important design consideration (in relation to the flexible membrane) is the operating stresses in the flexible membrane. The stresses in the diaphragm or disk are designed to be less than the endurance limit of the material used with some factor of safety.

The types of stress which must be considered are described in Appendix 2.

### How Diaphragm Couplings Affect the System

The forces and moments produced by a diaphragm coupling are caused by the axial and angular deflection of the metallic membrane.

1. *Axial force*—The axial force reacted back to the bearings is

$$F = K_A(d)$$

where $K_A$ is the axial stiffness of the coupling (lb/in.) and $d$ the axial deflection in inches.

2. *Bending moment*—The bending moment reacted to system shafts and bearings is

$$M = K_B(\propto)$$

where $K_B$ is the bending stiffness of the metallic membrane (in.-lb/deg) and $\propto$ the angle of misalignment in degrees.

### Safety Feature

Multiple diaphragms provide an additional safety feature. The stresses in each diaphragm in a pack are not equal due to the additional offset stress in the outer diaphragms. Therefore, if a failure does occur, it should occur first in the outer diaphragm. If the diaphragms are separated, one failed diaphragm will not necessitate the failure of the next diaphragm. When the stress component due to torque is less than the offset stress, the next diaphragm will actually have a lower combined stress than the failed diaphragm. This decreasing effect continues until approximately 50 percent of the diaphragms in the pack have failed. At this point the torque component of stress becomes larger than the offset stress. Therefore, a failure due to overmisalignment should be gradual and the diaphragm pack with a failed diaphragm will continue to operate. A failure could be detected audibly (a clicking noise) or the cracks may develop into broken pieces which would be trapped in the rigid hub or the guards. This may cause an unbalance which could be picked up on vibration monitoring equipment and give a warning to prevent a total diaphragm pack failure and possible damage to the connected equipment. A total failure could also be prevented by inspection of the coupling, at normal maintenance periods, for cracks or distress areas in the outer diaphragm of the pack.

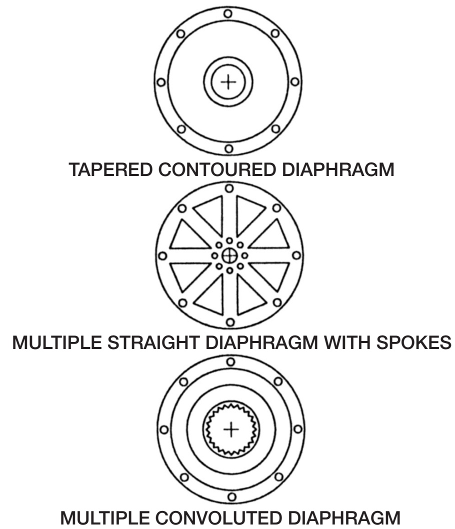



Fig. 23 Typical Diaphragm Couplings

### Comparison Between Gear Couplings and Diaphragm Couplings

Table 2 lists a comparison of high-speed, intermediate-speed, and low-speed diaphragm and gear couplings for a typical propulsion system. Coupling diameter, weight, torque capacities, bending moment, and axial force are listed.

The forces and moments are calculated for two different coefficients of friction to demonstrate the impact of this variation for gear couplings. The coefficient of friction will vary with the tooth design, type and quantity of lube, the types of material and tooth finish. For a diaphragm coupling the moments and forces are predictable and are simply related to the stiffness of the metallic membrane. Although a gear coupling will typically have a smaller diameter and a lighter weight for very high axial travel requirement applications, the bending moment imposed by the gear coupling on the system is still up to 35 times higher than the equivalent diaphragm coupling. The axial force imposed by the gear coupling is up to 4.5 times higher than the equivalent diaphragm coupling. For applications of up to moderate axial travel, the gear coupling may not only be heavier, but have five times higher

Table 2 Coupling Comparisons

| Coupling Type | O.D. (in) | Weight (lbs) | Continuous Torque Capacity (in-lbs) | Continuous Axial Travel (in) | Axial Force at Continuous Conditions (lbs) | Continuous Angular Misalignment (Degrees) | Bending Moment at Continuous Conditions (in-lbs) |
|---|---|---|---|---|---|---|---|
| High Speed Diaphragm | 22.88 | 300 | 500,000 | ± .625 | 1600 | ± .50 | 2000 |
| High Speed Gear (μ = .075) | 13.75 | 200 | 500,000 | * | 7500 | ± .50 | 73,000 |
| High Speed Gear (μ = .25) | 13.75 | 200 | 500,000 | * | 25,000 | ± .50 | 243,000 |
| Int. Speed Diaphragm | 22.88 | 135 | 1,000,000 | ± .250 | 4000 | ± .125 | 1800 |
| Int. Speed Gear (μ = .075) | 17.00 | 150 | 1,000,000 | * | 12,000 | ± .125 | 82,000 |
| Int. Speed Gear (μ = .25) | 17.00 | 150 | 1,000,000 | * | 40,000 | ± .125 | 273,000 |
| Low Speed Diaphragm | 81.00 | 42,300 | $40 \times 10^6$ | ± 1.25 | 35,000 | ± .375 | 230,000 |
| Low Speed Gear (μ = .075) | 74.00 | 53,000 | $40 \times 10^6$ | * | 83,000 | ± .375 | $3.4 \times 10^6$ |
| Low Speed Gear (μ = .25) | 74.00 | 53,000 | $40 \times 10^6$ | * | 277,000 | ± .375 | $11.3 \times 10^6$ |

* - Gear type couplings can be designed for unlimited axial capacities

axial force and 25 times the bending moment of the equivalent diaphragm coupling for the types shown.

Table 2 also compares calculations based on the requirements of MIL-C-23233 (coefficient of friction equals 0.25). The impact of these moments and forces on system design is even more significant and can require larger shafts, bearings, thicker flanges, etc., than with the dental coupling. For equipment that may not be impacted significantly such as pumps, the gear coupling can be a better choice for weight and cost.

For main propulsion couplings, reliability and maintenance considerations also show an advantage for the diaphragm coupling. Although gear couplings have been known to last 25 years or more, periodic sludge removal for continuously lubed couplings and replacement of the lubrication for sealed couplings are standard maintenance requirements. When couplings operate at low rotational speeds such as a lineshaft coupling, achieving adequate lubrication can require pressurized lube schemes. This is necessary because the centrifugal force from rotation may not be sufficient to fill an annular oil groove which submerges the teeth.

Another important feature of the diaphragm coupling is its noise characteristics. As the diaphragm coupling has lower stiffnesses, and transmissibility with no relative movement between parts, the noise attenuation of a diaphragm coupling for most cases will be better than the equivalent dental coupling.

## Summary

Both the gear coupling and the diaphragm coupling can be used in marine propulsion applications. The system designer must analyze the characteristics such as the reaction forces and moments produced by each type of coupling. The diaphragm coupling forces and moments are not only lower than a gear type coupling, but more predictable.

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# Appendix 1

## Gear Coupling Calculations

Compressive stress—axial sliding velocity criteria:

$T$ = torque, in.-lb
$PD$ = diameter of pitch circle of gear tooth, in.
$h$ = active tooth height, in.
D = diameter of curvature of tooth face, in.
$DP$ = diameter pitch of gear tooth
$N$ = maximum speed, rpm
= full-load misalignment angle, deg
$FW$ = gear tooth face width, in.
$R$ = PD/2, in.
$n$ = number of teeth in coupling
$C$ = percentage of teeth in contact

For crowned teeth (based on the compressive stress equation in MIL-C-23233):

$$S_c = 2290 \sqrt{\frac{T}{(PD/2) \times (D/2) \times c \times h \times (DP \times PD)}} = \text{psi}$$

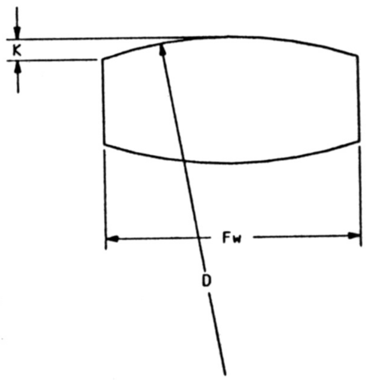


Fig. 24 Amount of tooth crown

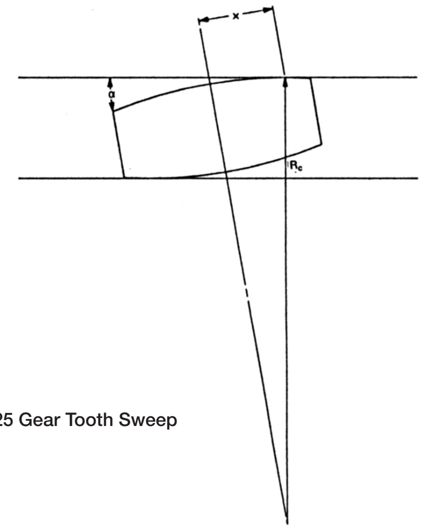


Fig. 25 Gear Tooth Sweep

If tooth curvature is expressed in amount of crown:

$$K = \text{amount of crown} = \text{in. (see Fig. 24)}$$

$$D = \frac{(FW/2)^2}{K} = \text{in.}$$

$$h = \frac{1.8}{DP} = \text{in.}$$

$$S_c = 3600\sqrt{\frac{T}{(PD)^2 \times D \times c}} = \text{psi}$$

For straight-tooth couplings:

$$S_c' = \frac{T}{RnA} = \text{psi}$$

$$A = \frac{1.8 \times FW}{DP}$$

Maximum sliding velocity ($V$) of gear coupling:

$$V = \pi \times PD \times \sin\alpha \times \frac{N}{60} = \text{ips}$$

The $S_C V$ criteria can be used to compare equivalent couplings and applications:

$$S_c V = S_c \times V = \text{psi} - \text{ips}$$

**Bending moment.** First, one must define a coordinate system for a gear coupling. In Fig. 18, the line $X'$ - $X'$ defines a cut through the plane of misalignment. Moments that tend to rotate the coupling around the $Z$ - $Z$ axis will be designated $M_z$. Moments that tend to rotate the coupling around the $Y$ - $Y$ axis will be designated $M_y$. $T$ is torque in in. lb.

Gear tooth sweep:

$$\sin\alpha = \frac{X}{R_c}$$

where $X$ is the distance from the centerline of the gear tooth to the point along the flank of the gear contact at misalignment (Fig. 25) and $R$, is the radius of curvature of gear teeth in inches.

Average radius for a misaligned coupling (Fig. 26):

$$R' = \sqrt{R^2 - \left(\frac{R}{3}\sin\frac{A}{2}\right)^2}$$

where $R$ is the pitch radius of the gear (in.) and A the angle for the gear tooth contact range in degrees.

The moments causing rotation around the z-axis:

$$M_z = \frac{T(\sin\alpha - \mu\cos\alpha)\sqrt{R^2 - [(R/3)\sin(A/2)]^2}}{R(\cos\alpha - \mu\sin\alpha)}$$

Moments causing rotation around the $Y$ - $Y$ axis:

$$M_y = \frac{M_t X}{R}$$

Resultant moment is the vectorial combination of two moments:

$$M_2 = \sqrt{(M_z)^2 + (M_y)^2}$$

at approximately from $Y$ in the $ZY$-plane.

Final combined moment equation:

$$M = \sqrt{\left[\frac{T(X)}{R}\right]^2 + \left[\frac{T(\sin\alpha + \mu\cos\alpha)}{R(\cos\alpha - \mu\sin\alpha)}(R')\right]^2}$$

**Reaction forces on equipment bearings.** Since maximum moment occurs at $\theta = \tan^{-1}(2R/Dc)$ from $Y$ in the $ZY$-plane, the maximum bearing loads are perpendicular. Pictorially this is difficult to show, so the loads will be shown as if they are parallel to the $Z$-axis. This does not affect the magnitude of the load, but only the point of application, which is not very important. The bearing reactions will be solved in terms of maximum moment, as if the maximum moment were tending to rotate the coupling around the $Y$ - $Y$ axis.

Considering Fig. 17 (plan view), with torque and rotation directions shown and a plane of misalignment as given, it is observed that the force pattern nets a clockwise

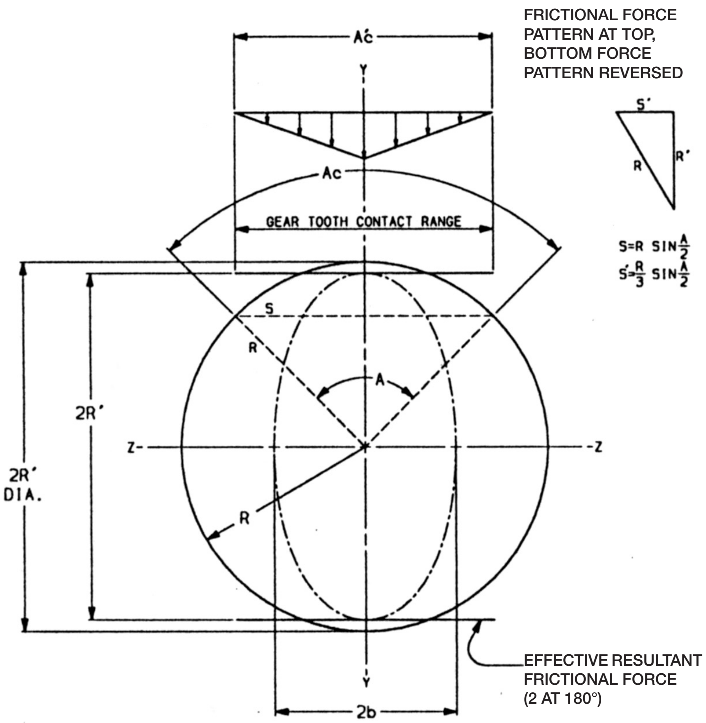


Fig. 26 Average Radius for a Misaligned Gear Coupling

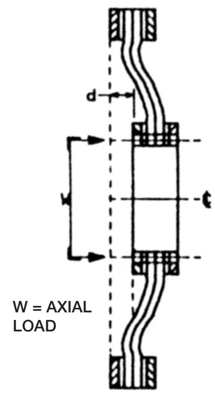


Fig. 27 Axial Deflection of Diaphragms

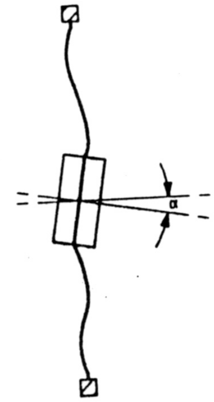


Fig. 28 Angular Flexure of a Single Diaphragm

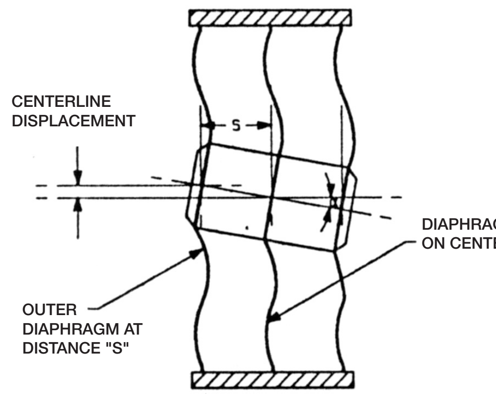


Fig. 29 Deflection in Diaphragms Due to Offset

moment transferral (*Mft*) to the shaft. The bearing nearest the transferral moment would then have a high-pressure condition, *B*, in the *Z*-plane, which loads the front bearing down and the back bearing up. This assumes the direction of rotation to be over the top of the shaft.

Taking the *M*-values about *A*, and setting them equal to zero, will give

$\Sigma M_A = B'L - M = 0$
$B' = M/L$ resisting force required of bearing
$b' = A'$ equal and opposite in direction

The resulting force pattern for *A* and *B* would then be the opposite of *C* and *D*, reading from left to right. Figure 17 indicates the force pattern that results for the direction of rotation as shown. Conversely, when the direction of shaft rotation is reversed with the same misalignment condition, the forces reverse themselves as indicated.

## Appendix 2

### Diaphragm Stresses

As an example, a multiple convoluted diaphragm will be used, but the approach is very similar for most flexible membrane couplings as most have the same type of stresses.

To understand diaphragm stresses it is important to understand how the diaphragm reacts to the various types of misalignment. Figures 27, 28 and 29 are exaggerated in order to demonstrate more clearly the diaphragm reaction to various forces. It is also important to realize that a diaphragm on the centerline of the diaphragm pack reacts differently than a diaphragm off the centerline, when angular misalignment is imposed on the entire pack.

Some of the stresses resulting from the diaphragm deflection shown are continuous during the entire period of operation, and these are termed "steady state." On the other hand, some of the stresses not only vary, but go through complete reversals during each revolution. These are termed "alternating."

**Steady state stresses.** The steady-state stresses are considered to be the stresses which result from axial displacement of the diaphragm (Fig. 27), torque transmission, centrifugal effects, and thermal gradient effects.

**Axial stress ($S_A$) (Fig. 27)**. ($S_A$) is determined by the amount of deflection imposed on the diaphragm. *W* is axial force, *d* is distance moved. The stiffness of a pack is equal to: $K_a = W/d$. The stiffness of a coupling is equal to: $K_A = W/2d$.

**Shear stress (τ)**. (τ) occurs when torque is transmitted through the diaphragm pack and is dependent upon the size, number and thickness of the diaphragms. Shear stress (τ) is highest at the inside diameter of the diaphragm. NOTE: In cases where torque is cyclic or reversing, this stress or part of it must be combined with other dynamic stresses.

**Centrifugal stress ($S_c$).** ($S_c$) always results when the coupling is rotated and this rotational effect on the diaphragm must be combined with the other steady-state stresses.

**Thermal gradient stress ($S_t$)**. ($S_t$) applies only where there is a temperature differential across the surface of the diaphragms and/or where there is a coefficient-of-expansion difference. If this case exists the combined thermal stress must be calculated.

**Alternating stress ($S_B$).** Two cases are considered which contribute to the total alternating stress: angular flexure stress (Fig. 28) and stresses due to offset deflection (Fig. 29).

**Flexure stress ($S_F$). (Fig. 28**). ($S_F$) is the result of the angular misalignment of the coupling, ( $\propto$ ) is misalignment angle in degrees.

**Offset stress ($S_0$) (Fig. 29)**. ($S_0$) is caused by angular deflection of the inside diameter of the diaphragm with respect to the outside diameter. The diaphragm elements which are spaced axially away from the centerline of flexure experience a stress proportional to the distance that they are removed from the centerline. Figure 29 shows a cross section of an exaggerated pack and how the outer diaphragms are compressed or stretched due to the distance(s) removed from the center of misalignment.

**Combined diaphragm stress.** All the stresses presented above are calculated at the inside diameter of the diaphragm and for the farthest diaphragm from the centerline and are then combined in the following manner to give the highest stress point in the diaphragm pack. Axial stress, thermal stress and centrifugal stress are additive. Total steady-state normal stresses ($S$) are computed as follows:

$$S = S_A + S_t + S_c$$

These are then combined with the shear stress to produce the combined steady-state stress ($S_M$):

$$S_M = \frac{S}{2} + \sqrt{\left(\frac{S}{2}\right)^2 + \tau^2}$$

Total alternating stress ($S_B$) is conservatively determined by the simple summation of the offset and flexure stresses. Where no cyclic torque is present:

$$S_B = S_0 + S_F$$

If cyclic torque is present:

$$S_B = \frac{S_0 + S_F}{2} + \sqrt{\left(\frac{S_0 + S_F}{2}\right)^2 + \tau^2}$$

Finally, the mean stress and the alternating stress resulting from bending of the diaphragm are combined by the Soderberg equation. MIL-C-23233 for Navy propulsion coupling applications requires a minimum safety factor of 2.0.

$$\frac{1}{N} = \frac{S_M}{S_{yld}} + \frac{S_B}{S_{end}}$$

where

$N$ = Safety factor

$S_{yld}$ = Yield strength of material, psi

$S_{end}$ = Endurance strength of material, psi

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